

**NASA Technical Memorandum 85741**

NASA-TM-85741 19840008523

**STRUCTURAL OPTIMIZATION: CHALLENGES AND OPPORTUNITIES**

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**JANUARY 1984**

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# STRUCTURAL OPTIMIZATION: CHALLENGES AND OPPORTUNITIES

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## ABSTRACT

Recent developments in structural optimization, when taken collectively, promise informed practicing engineers a quantum jump in their design capability. In this paper, the area of structural optimization is treated in the broader context of a vehicle design process with a focus on structural sizing. A basic introduction to a formal approach is given, and several application examples are illustrated, to lay a background for the review of recent progress. The main developments discussed include techniques for reducing computational cost of optimization, methods for generating sensitivity information, and the ways to make the computer implementations more practical. New prospects are presented for applying optimization to very large problems by formal decomposition into a number of smaller problems in a manner compatible with the trend toward distributed computing for the design process organized into specialty groups. Numerous references are quoted as points of entry to the vast literature on the subject.

## INTRODUCTION

This paper's purpose is to alert engineers to several recent developments in structural optimization which, collectively, offer a new, exciting capability that should result in a quantum jump of their productivity when incorporated in design practice. To that end, we take a bird's eye view of structural optimization as a tool for the designer, examine the trends further developments are likely to follow in the near term, and extrapolate these trends into the future.

To begin with, it may be illuminating to take a broad look at the entire design process in which optimization may be used. A generic form of such a process applicable to any engineering product is shown in figure 1. It begins with a functional definition of the object to be created, and then moves on through the phase of evaluation of the external influences (e.g., loads on a structure) to the selection of design concept, material, overall geometry and internal layout.

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N84-16 591 #

Quantification of the design physical characteristics is next, followed by a detailed design to be passed on to a shop that will give the object its material form.

Since the ways in which the product will be fabricated, tested, and maintained must be considered in its design, the corresponding phases are shown in figure 1 as integral parts of the entire conception-to-scrap-heap cycle.

An important part of the process is the feedback (symbolized in figure 1 by arrows and a bar to the right) which renders the whole process iterative. In general, the iterations are numerous, may span various parts of the process, and may be nested in several levels.

The process laid out in figure 1 illustrates a course of action engineers have been taking successfully for centuries, enjoying its creative aspects while enduring its drudgery. In recent times, they have been receiving more and more support from the computer in all phases of the process as shown in figure 1 by the bar to the left.

Basically, computer support consists of data storage and retrieval, graphics, fast analysis with multipath logic, and the capability to interface with external sensors and to drive the external devices (e.g., numerical control of production machinery). These various forms of support are utilized as shown by arrows in figure 1. Consistent with the nature of man and computer, the creative part of the process tends to stay with the former while the latter is gradually taking over the drudgery.

In this whole picture, our initial focus will be on an important but relatively small part: optimization of the quantitative characteristics of the design. In that small part, we will concentrate on a still smaller fragment: the optimization of structural member cross-sectional dimensions (asterisk in figure 1) to minimize structural mass. This will serve both as an introduction to the subject and as a basis from which to review the recent advances and project into the future.

## FORMAL OPTIMIZATION IN STRUCTURAL DESIGN

Probably, the earliest method for structural optimization has been a Fully Stressed Design method which iteratively modifies a cross-sectional area by the ratio of member stress to allowable stress. This time-honored method gave rise to a whole body of weight-strength algorithms (ref. 1)\* and, later, to a class of methods known as optimality criteria methods (ref. 2). Another class of methods useful in structural optimization originated from the optimal control theory (ref. 3). This report deliberately excludes these classes of methods in order to focus on the methods based on nonlinear mathematical programming (NLP).

### Introductory Example

A tubular column, figure 2, is a simple example to establish the basic definitions and concepts. Let's assume that at earlier stages of design, it was decided that a weight  $P$  has to be supported at a height  $z$ , and that a tubular column is to be used for the support. These were the qualitative decisions made upstream of the design process. What is left now is a quantitative decision of sizing the cross-sectional design variables of  $R$  and  $t$  for a minimum objective function of the column mass, achievable within constraints of stress, buckling, and minimum gages.

Under the NLP formalism, the problem is reduced to finding a minimum of the objective function constrained by the inequality constraints in a

\*The references are listed at the end in consecutive order.

design space defined by the design variables. In the simple problem at hand, the space is two dimensional, and the problem can be graphed in the  $R, t$  coordinates as shown in figure 3. Each curve in the figure represents a constraint boundary (labeled by the corresponding constraint, for example: an allowable stress constraint) and divides the design space into domains which are feasible and infeasible (cross-hatched side) with respect to that particular constraint. Superposition of the objective function contours (the dashed curves) on the constraint boundaries reveals point  $C$  as the constrained minimum of mass.

#### Simplicity of the Concept

Optimization based on nonlinear mathematical programming (NLP) clearly separates the analysis from search in the design space and leads the computer implementation to a scheme shown in figure 4. In that scheme, the module labeled "optimizer" is a program searching the design space for a constrained minimum following the numerical information on the objective function and constraints supplied by the "analyzer" which carries intelligence about the physics of the problem. The iterative execution of the optimizer and analyzer in an optimization loop is stopped by the "terminator" when appropriate mathematical and physical convergence criteria are met. The scheme in figure 4 is the simplest one possible and, as later discussion will show, it becomes more complex in large-scale applications.

#### State-of-the-Art Applications

There have been numerous optimization applications during the past 2 decades. Some recent examples range from components of automotive structures (refs. 4 and 5) to entire aircraft structure (ref. 6). The latter is depicted in figure 5 whose inset shows the mass reduction in the process of optimization. A more general application involving performance of an entire system describes optimization of a light aircraft for a minimum cost of ownership (ref. 7).

Recent surveys (refs. 8, 9, 10, 11, 12) present hundreds of similar examples showing development of optimization methods which is both rapid and accelerating as attested to by 177 references cited in reference 11 alone. However, reference 11 also revealed that most applications are test cases carried out to verify methods under development rather than applications in real design.

In the next section, we will examine the methods and techniques that have recently become available, to show that a coalescence of these new capabilities and the needs of industry for increased productivity creates a new environment in which the application lag indicated in reference 11 may be eliminated.

#### RECENT DEVELOPMENTS

Each of the particular methods, techniques, and algorithms discussed in this section has been selected on the basis of their special potential for making optimization methods more cost effective and easier to apply from a practicing engineer viewpoint, and as being either at or near the maturity level required of the tools ready for industrial use.

#### Search for Constrained Minimum

A recent survey (ref. 13) provides an efficiency rating of 14 algorithms for searching design space for constrained minimum that are candidates for the "optimizer" function in figure 4. Since it is very difficult, if not impossible, to distinguish in such efficiency comparisons between the efficiencies of the optimizer and the associated problem-dependent analyzer, results of this and other similar surveys have to be taken with a grain of salt and one has to acknowledge that no

consensus has, as yet, emerged on how to order the list of algorithms. However, two algorithms known as an Augmented Lagrangian Method and a Generalized Reduced Gradient Method have gained recognition as leading contenders for top spots on the list. Both methods, whose mathematics is explained in the literature (for example, refs. 14 and 15) are credited with fast convergence and have an important practical advantage of being able to begin with either feasible or infeasible designs.

Before leaving this subject, one might add a note of caution that there are problems in constrained minimum search that still can only be solved for a limited class of applications or that pose enormous numerical difficulties due to complex and discontinuous shapes of the constraint boundaries. An example of the former is a multiple minimum problem (ref. 16) and an example of the latter is optimization of a structure with dynamic constraints (ref. 17).

#### Computational Cost Reduction

Most of the cost in large-scale problems, in fact as much as 99 percent of it, stems from repetitive execution of analysis. Radical reduction of that cost, therefore, can best be achieved by reducing the use of full analysis in the optimization loop and using an inexpensive but approximate analysis in the loop as depicted in figure 6. There are several techniques available that may be used for approximate analyses: they are grouped in categories in Table 1. All the categories have a common goal of providing means for rapid (i.e., inexpensive) analysis of a modified structure.

Extrapolation methods. One of the two main approximation techniques is a linear extrapolation (top of the table) by a Taylor series (refs. 18 and 19) using first derivatives, or by a perturbation technique (ref. 20) akin to the small parameter method used for the solution of nonlinear differential equations. The usefulness of linear extrapolation in optimization (known as a piecewise linear optimization) has been demonstrated many times; a particularly convincing example is given in reference 21.

Dimensionality reduction. The other main approximation technique is a dimensionality reduction which reduces the number of unknowns in the equations to be solved repetitively in the optimization loop. There are several techniques in this major category. The lumping technique results in two finite-element models: a highly refined one used in the full analysis (loop \$\$ in figure 6) and a simplified model, used in the approximate analysis (loop ¢¢) and periodically adjusted to correlate its characteristics with the results from the refined model. This approach proved to be useful for thin-walled structures such as a delta wing case (ref. 22) where large built-up subassemblies may be represented effectively by stiffness- and mass-equivalent plates and beams. The well-known method of substructuring, e.g., (ref. 23) has the same effect of simplifying the finite-element model. A particularly useful form of substructuring, called a superelement method is described in references 24 and 25.

The reduction of the number of unknowns in the analysis can also be obtained by formal condensation, e.g. (ref. 23), or by a Rayleigh-Ritz method using appropriate base functions. It has been shown in reference 26 that the resulting condensed stiffness and mass matrices can be very inexpensively updated in the optimization by a linear extrapolation, if the base functions can be regarded as constant. Dimensionality reduction in analysis can also be carried out by an incomplete modeling (ref. 27) and by multigrid techniques (ref. 28). In response to a similar need for reduction of the repetitive analysis cost in nonlinear structural

analysis, a number of useful techniques have been developed as reviewed in reference 29.

Dimensionality reduction in optimization is not limited to just the reduction of the number of unknowns in analysis; it also entails reduction of the number of design variables (ref. 30) and repetitively evaluated constraints (ref. 19). The design variables can be reduced in number by making them dependent on a smaller number of judiciously chosen "master" variables. By means of such variable linking, designers can effect their judgment (as they do now) as to the way cross-sectional dimensions should be distributed over the structure.

Primal-dual approach. Another approach to the analysis cost reduction which now appears to be developing into an entirely distinctive body of techniques, is known as primal-dual method whose comprehensive exposition can be found in references 31 and 32. The method maps the original structural optimization problem (primal problem) from its design space of many physical design variables into a space defined by the problem's Lagrange multipliers. The substitute problem (dual problem) in that space has a relatively much smaller number of variables because the number of Lagrange multipliers is equal to the number of active constraints, which is usually much less than the number of physical design variables. In addition, the only constraints in the dual problem are arithmetically very simple requirements of non-negativity of the Lagrange multipliers. The optimum solution of the dual problem is mapped back into the space of the physical design variables to obtain the original problem solution. The three-phase operation of mapping from primal to dual space, dual problem optimization and reverse mapping is iterated because the mapping relies on approximate explicit relations based on derivatives of the objective function and constraints with respect to the physical design variables, and these derivatives need updating as the design moves away from the point where they were evaluated. Besides good convergence, the method offers a very important benefit of being able to handle discrete design variables (e.g., standard thicknesses of sheetmetal) as well as continuous ones within the same problem (ref. 33). It also has a theoretical significance of unifying the optimality criteria and NLP methods as shown in reference 34.

Computation of derivatives. Most of the previously discussed techniques depend on derivatives (gradients) of the objective function and constraints with respect to the design variables. In many practical applications, the cost of computing these derivatives by finite differences may become prohibitive, despite placing the computation outside the optimization loop, therefore, analytical techniques for derivative calculation are preferred.

As a general principle, the derivatives of a numerical solution can be obtained analytically by differentiating the governing equations of the problem in order to construct equations which contain the derivatives as unknowns. For example, differentiation of the load-deflection equations  $Ku = L$  with respect to a design variable  $x$  yields the equation  $K(\partial u / \partial x) = (-\partial K / \partial x) u$ , for  $\partial P / \partial x = 0$ , that can be economically solved for  $\partial u / \partial x$  at a cost similar to that of another loading case by reusing the decomposed matrix  $K$  and the vector  $u$  saved from the solution of  $Ku = L$ . The derivative  $\partial K / \partial x$  can be inexpensively obtained, either analytically, if  $K$  is a linear function of  $x$  (e.g.,  $x$  being a membrane element thickness), or by finite differences. Use of the latter in the analytical equation for  $\partial u / \partial K$  is referred to as a semianalytical technique for computing  $\partial u / \partial x$ .

Algorithmic details for analytical and semianalytical technique are given in many sources (e.g., refs. 5, 26, 35, 36), and, recently, a method has been proposed in reference 3 for bypassing the stiffness

matrix derivatives in calculation of derivatives with respect to the overall shape variables. Some examples of the efficiency of analytical and semianalytical techniques in application to aircraft type, built-up structures are given in figure 7 taken from reference 37.

The general principle underlying the analytical derivative calculation is also valid, if the governing equations are solved iteratively and, therefore, it applies in dynamic and buckling analysis for derivatives of eigenvalues and eigenmodes (refs. 5, 38, 39). It also generalizes to the higher order derivatives (refs. 40, 41, 42).

As pointed out in reference 5, the sensitivity information in the form of derivatives is useful to designers as a quantitative guide and an answer to the "what if" type of questions, even if it is not used in an optimization algorithm. As evidenced by references 35, 37, and 43, one may expect that sensitivity information will soon become an output option routinely available in major structural analysis programs.

### Design-Oriented Analysis

Developments discussed in the preceding subsections have evolved collectively into a new capability that can be called design-oriented analysis. Its basic features over and above regular analysis are a capability for gradient generation, means to trade the computational cost for accuracy (e.g., selection of techniques from Table 1), and a modular implementation that keeps outside the optimization loop those parts of the analysis that remain unaffected by the design variable changes. Experience to date indicates that by use of design-oriented analysis, the computational cost of structural optimization can be well controlled and kept below 30, and in many cases below 10, times the cost of the ordinary analysis of the structure at hand.

Once the optimization cost is under control, it is possible to compare that cost with the benefits of the reduced manpower cost, shortened task calendar time, and increased design quality. In this manner, one can rationally assess the optimization cost effectiveness in a particular application at hand.

### Optimization in Structural Dynamics

Applications to structures under dynamic loads which are particularly important to automotive design have been impeded by the phenomenon of resonance which splits the design space into disjoint subspaces. Since most of the search algorithms require continuity of the functions and their derivatives, these algorithms can not search across the disjoint subspace boundaries, and because the number of subspaces grows rapidly with the number of eigenvalues it is impractical in most cases to perform a separate search within each subspace.

An innovative solution to that difficulty has been recently offered in reference 44. Under the proposed approach, the problem is divided into a continuous optimization performed within the resonance bounds in one subspace at a time, followed by a transfer to a subspace inexpensively selected from the neighboring subspaces on the basis of having the greatest potential for further structural mass reduction.

### Sensitivity of Optimum Designs to Problem Parameters

From a designer's viewpoint, existing optimization techniques have a deficiency of providing single point information, when compared with the traditional parametric studies that clearly present "what if" type of information in a graphical form. That deficiency can be eliminated by analyzing an optimum solution for its sensitivity to those physical quantities that were kept constant (problem parameters) in the optimization. For example, assume that the cantilever truss shown in figure 8 was optimized to obtain minimum structural mass and optimum cross-sectional areas subject to stress constraints. The truss dimensions  $H$  and  $L$



were among the constants of the problem. Now we want to know how the mass and cross-sectional areas change when the dimension  $H$  is increased by, say 10 percent. Intuition fails to give the answer because there are two conflicting trends: mass of the horizontal rods can be reduced as the truss depth increases, but the upright and diagonal rods gain mass since they become longer. The answer can be obtained without the "brute force" parametric approach of repeating the optimization for an incremented  $H$  by means of an algorithm described in reference 45.

The algorithm is based on the same approach of differentiation of the governing equations that was discussed in the section on computation of derivatives. For a constrained minimum, the governing equations are the Lagrange multiplier equations whose differentiation with respect to the parameter leads to a set of simultaneous algebraic equations. In these equations, the coefficients are functions of the objective function and active constraint derivatives evaluated at the constrained minimum, and the unknowns are the derivatives of the optimum design variables with respect to the parameter. Solution of the equations, produces the derivatives which are then used to calculate the total derivative of the objective function.

In the truss example, the optimum sensitivity derivatives with respect to  $H$  can be used to extrapolate as shown in figure 9 to find that the mass actually decreased to 0.97 of the reference minimum mass for 10 percent increment of  $H$ . Reoptimization confirms the above result and indicates (continuous line in figure 9) that in this case the linear extrapolation's accuracy is reasonably good for up to 20 percent change of  $H$  for structural mass and for cross-sectional area of rod 1 (taken as an example of a design variable). Although introduced in the context of structural optimization, the algorithm is entirely general. It could be used, as well, to evaluate the sensitivity of the maximum payload to the aircraft range assumed as a problem parameter during the optimization of the aircraft. A similar application in automotive design might be the sensitivity of the minimized fuel consumption per unit of distance to the parameter of a minimum required acceleration.

#### Adaptability of Software to Variety of Design Applications

The task of developing an optimization program system equipped with the search and analysis capabilities suitable for the present and future needs of a large number of users is more difficult than development of a specific analysis program. The reason is that design, in contrast to analysis, is to a large extent an art and we want it to remain that way least we lose its vital ingredients of creativity and inventiveness. This puts an optimization software developer in the situation of not knowing, precisely, what will be the objective function, design variables, and constraints in each potential application and what will be the preferred procedure. The use of a number of prewired options, similar to the NASTRAN (ref. 46) "rigid formats," does not allow for the required flexibility, and past attempts to use this approach resulted in optimization programs that were often disappointing to the users.

One solution lies in a programming system concept whereby the problem dependent parts of the procedure are left for the user to code in the form of programs (usually simple and short) to be inserted in the system when the optimization problem is formulated. An example of such a system is PROSSS (for Programming System for Structural Synthesis), which is described in references 47, 48, and 49. In PROSSS, the problem-dependent codes connect the analyzer and optimizer as seen in figure 10. The Optimization-to-Analysis (O-A) processor is a code that embodies the design variable definition. It converts the numbers the optimizer manipulates as design variables to the structural input parameters recognized by the analyzer. All sorts of judgmental devices such as design variable linking (ref. 30) can be coded into this processor. The

Analysis-to-Optimization (A-O) processor on the opposite side of the analyzer incorporates the objective function and constraint definition. It selects from the analyzer's output the quantities needed to calculate the objective function and constraint values in a manner decided by the user. The flowchart in figure 10 is the simplest of the PROSSS procedural options and corresponds to figure 4. Other options include combinations of an analytical generation of gradients and a piecewise-linear optimization.

The programming system approach permits a common core of programs, termed a skeleton form in figure 11, to be shared by many forms specialized for applications (right hand side of figure 11). Each specialized form has its own O-A and A-O processors which, once prepared for a certain class of applications, can be assembled with the skeleton modules for execution using executive software generally available on most computers in the form of utilities embedded in the operating system. If he so desires, a designer can limit his view of the programming system prepared for a specific application to see it as a black box into which he feeds input to obtain optimum solutions. The details of setting up the specialized forms for the variety of company applications can be left to staff specialists who can replicate the system into an array of specialized tools.

Two examples of very different applications of one programming system from reference 48 are shown in figures 12 and 13. The former illustrates a minimum mass optimization of a fuselage segment under stress, displacement, and buckling constraints with the inset showing the resulting cross-sectional material distribution. The latter depicts a shape optimization that starts with a portal framework and ends with a significantly lighter truss.

### Large-Scale Problems

Many practical structural design problems are so complex and contain such a large number of design variables and constraints that it is impractical to try to optimize them as single problems. There are also organizational difficulties. If the problem is large, it is certain that a design organization is, or will be, created to allow many engineers and, preferably, many computers to work on various parts of the problem concurrently. This is the sound management principle of developing a broad work front in the organization. The concept of folding that work front into a single stream within an optimization procedure does not appear to be a very practical proposition. However, an optimization procedure can be developed that allows for a systematic decomposition of the problem into several parts to cover the widely developed organizational work front. In operations research, there are mathematical means to carry out such decomposition (ref. 50), primarily in application to economic systems. A formulation of decomposition for engineering systems is given in reference 51 which shows how to preserve the couplings between the parts of the decomposed problem by means of the optimum sensitivity derivatives discussed previously. A special case of decomposition of a large structural optimization problem leads to an optimization procedure that parallels the process of analysis by substructuring. One particular form of such decomposition has been proposed in reference 52. A somewhat different implementation is shown in reference 53.

Two-level decomposition. The basic concepts of the decomposition process can be presented in a simplified manner by considering a two-level approach. A minimum mass optimization of the simple framework shown in figure 14 (ref. 53) will serve as an example. Without decomposition that optimization could be carried out with the 18 cross-sectional dimensions depicted in figure 14 (inset) as design variables

and with constraints imposed on the displacements, and on stresses due to the material limits and local buckling. In optimization by decomposition, this problem is divided into a set of three separate subproblems for each beam and a coordination problem for the assembled structure. In a beam subproblem, the beam is regarded as loaded by the invariant end-forces obtained from the assembled structure analysis, and the design variables are six detailed cross-sectional dimensions. The objective function is not the beam mass. Instead, it is a cumulative measure of the stress and local buckling constraint violations in the beam which is being minimized subject to inequality constraints in form of the bounds on the dimensions, and the equality constraints that make the beam cross-section area,  $A$ , and moment of inertia,  $I$ , equal to the values set in the coordination problem.

In the coordination problem, the framework structure is optimized for minimum mass subject to constraints on the framework displacements and the cumulative measures of constraint violations in each beam using the beam  $A$ 's and moments of inertia  $I$ 's as six higher level design variables. In this optimization, the cumulative measures of constraint violation are linearly extrapolated for each beam using their optimum sensitivity derivatives with respect to the end-forces and the beam  $A$ 's and  $I$ 's which are parameters in the beam optimization subproblem. The coordination problem (which includes the assembled structure analysis), and the beam subproblems (which can be worked on concurrently), are repeated iteratively until the objective function is minimized and all constraints are satisfied. The two-level approach is illustrated in figure 15 which shows the decomposition tree and the information exchanged between the structure and substructure levels. Transmitted downward are the values of  $A$ ,  $I$ , and the end-forces for each beam. Returned upward are the derivatives of the cumulative measures of constraint violations to convey to the system level the coupling information how the beam constraints will react to changes in the values of  $A$  and  $I$ . The result is a conversion of a problem of 18 design variables into 3 problems of 6 local variables each and 1 problem of 6 higher level variables. Comparison of the approach with the optimization without decomposition reported in reference 53 showed good correlation of the results and promising overall effectiveness of the decomposition method.

Multilevel decomposition. Considering structural assemblies much larger than the framework example, we can envision figure 15 expanded to a tree of more than two levels as shown in figure 16 for an aircraft structure. Each of the elements or nodes of this scheme could be occupied by groups of people and computers working concurrently on their parts of the problem. In other words, we see the decomposition scheme blending with the familiar organization of a design office into a wide work front for people and machines.

#### Computer Power

Recent computer trends toward cheaper hardware, and distributed computing replacing centralized processing support very well the optimization developments presented herein. Already available are electronic work stations interconnected in a network (ref. 54) that give an engineer the desk-side power of a large computer, an access to computing and data handling capability of any computer in the network, and, what is most important, a direct access to data generated by other members of the design team. High-level programming languages are also available for executing major computer codes in logically complex sequences, as if they were subroutines called from a main program, even though they may run on different computers. A step in that direction was

taken by dispersing the programming system from reference 48 between a minicomputer and a mainframe computer (ref. 55).

Optimization procedures tend to generate voluminous data and, therefore, can benefit from the new software for efficient data handling that have recently been developed especially for engineering purposes (ref. 56). That software addresses the data management problem at two levels. The lower level program (ref. 57), an equivalent of a file cabinet, has already been used with excellent results in major applications (ref. 58). The higher level program (ref. 56), an equivalent of a reference library, is in testing.

It is apparent that these computer technology developments not only form a base on which to operate the organization shown in figure 16 but they actually require such organization if their full potential is to materialize.

#### CONCLUDING REMARKS

Recent developments in structural optimization techniques and increases in computer power have been reviewed. The picture emerging from the review can be summarized as shown in figure 17. Several developments, each corresponding to a section in this paper, have a great potential to reinforce each other with a synergistic result of a quantum jump in practical usefulness of optimization in design. The improved optimization capability coincides with a steadily increasing need for formal methods to support design of structures in which new technology, such as composite materials, or new functional requirements, such as a significantly lower vehicle mass for improved fuel efficiency, reduce the reliability of previous experience in guiding the design decisions, especially the quantitative ones. It is apparent that all ingredients now exist to spur the expansion of optimization methods from their present limited position in figure 1 both upstream and downstream of the design process. Upstream, the challenge of multidisciplinary optimization will immediately be encountered and the rewards will be magnified because each discipline will have an opportunity to influence the design earlier than is now feasible. Downstream, the role of optimization as a mathematical organizational framework for a large team will be increasingly important bringing benefits commensurate with the large resources committed during the final stages of the design process.

However, the synergistic use of all the methodology reviewed in this presentation in everyday design practice is unlikely to materialize soon unless a concentrated effort is made by practitioners, managers, and researchers who are motivated by an awareness of the opportunities and potential gains offered by the evolving state of the art. If this presentation contributed to that awareness, its purpose will be fulfilled.

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Table 1.- Techniques for design oriented structural analysis.

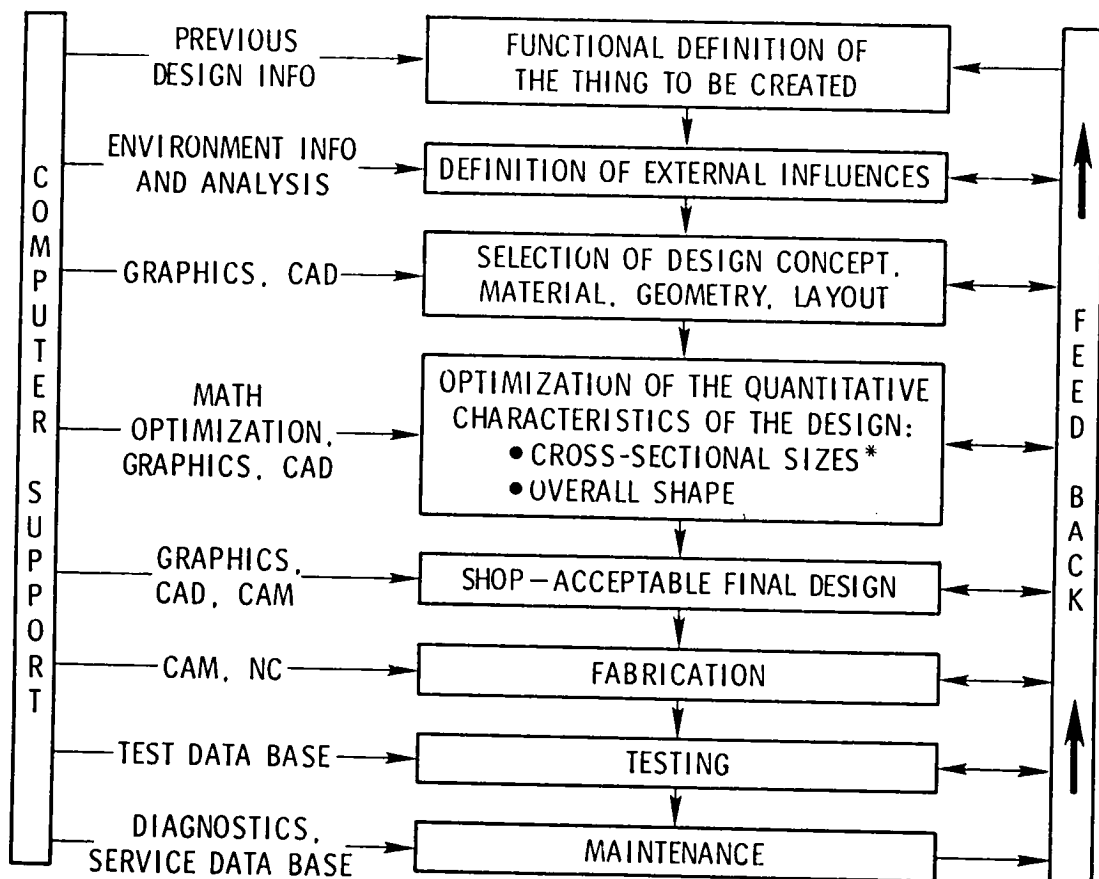
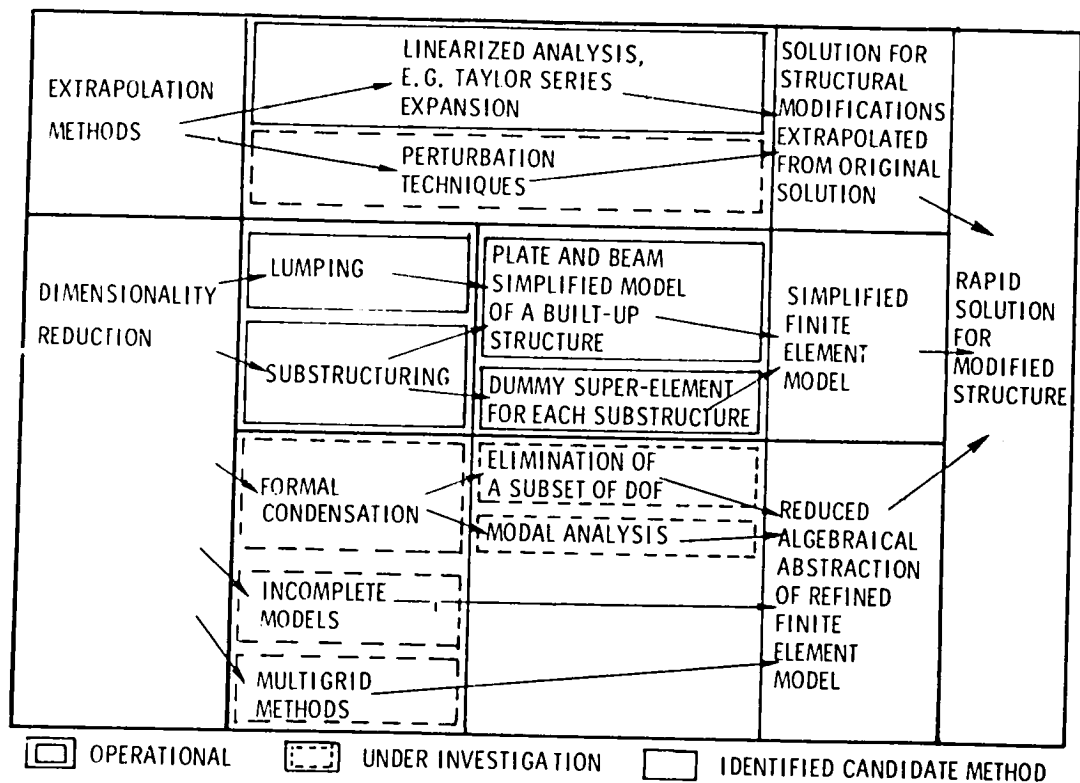


Fig. 1.- A design process.

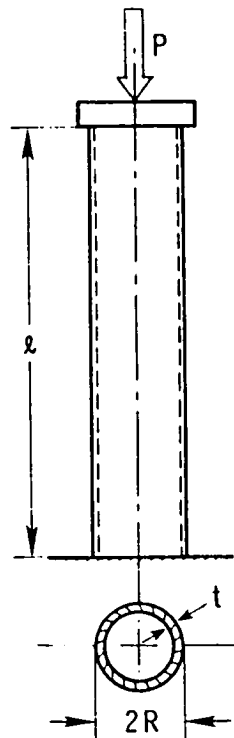


Fig. 2.- Compressed column as an example of structural optimization.

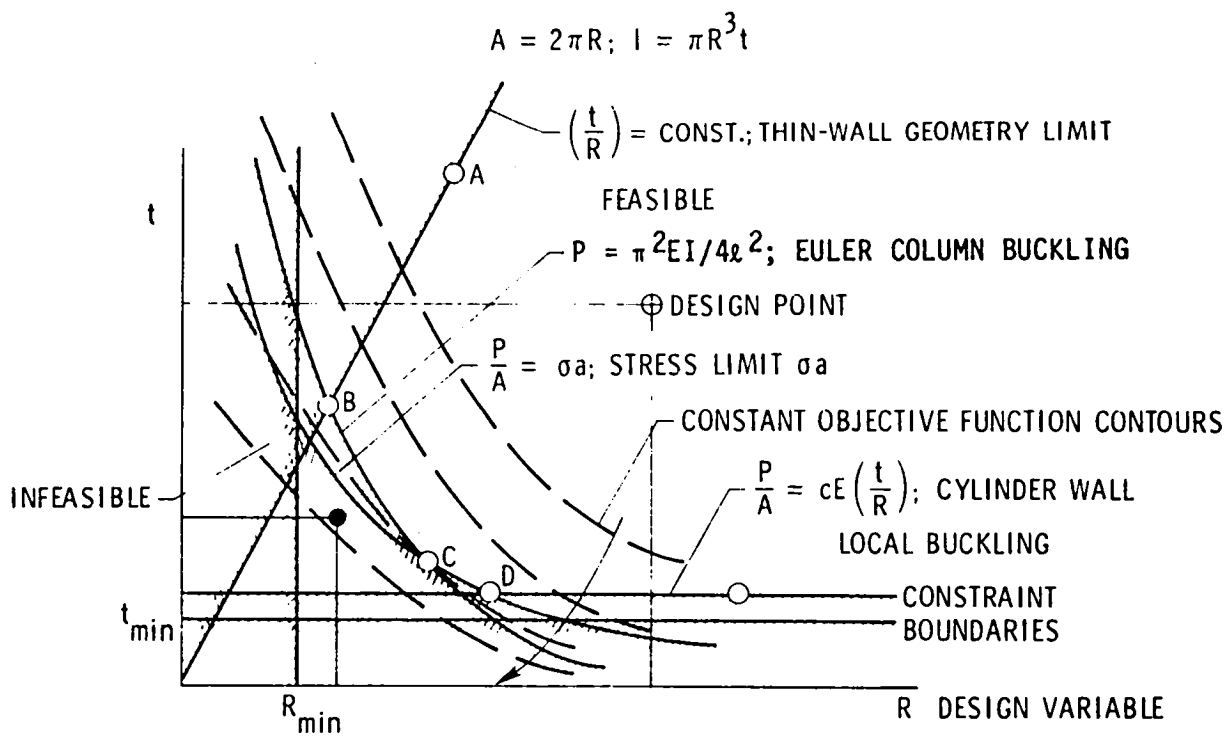


Fig. 3.- The column design in a constrained design space.

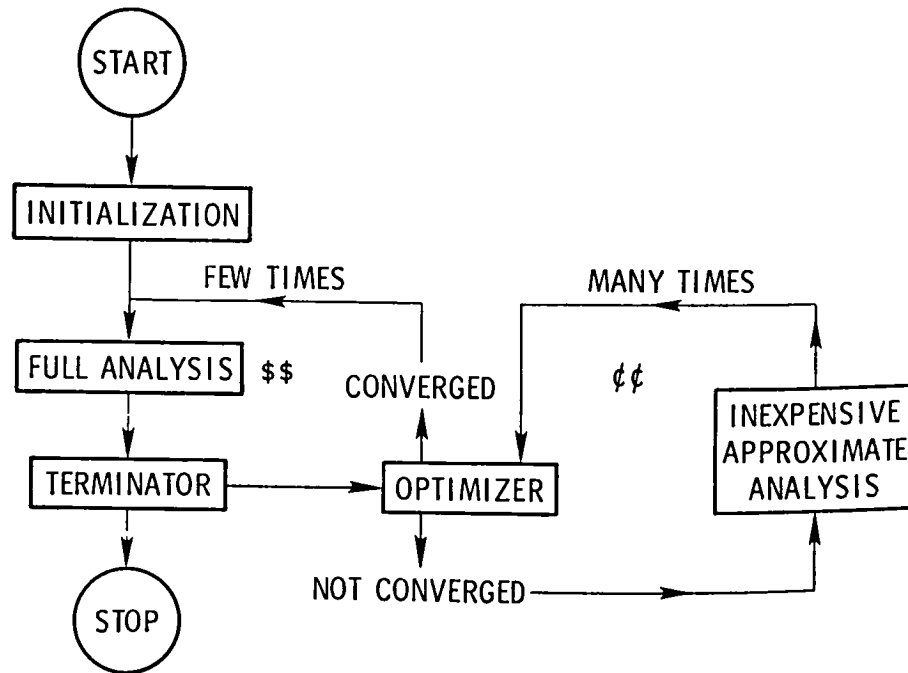


Fig. 6.- Flow chart of an optimization procedure with the full analysis placed outside the optimization loop.

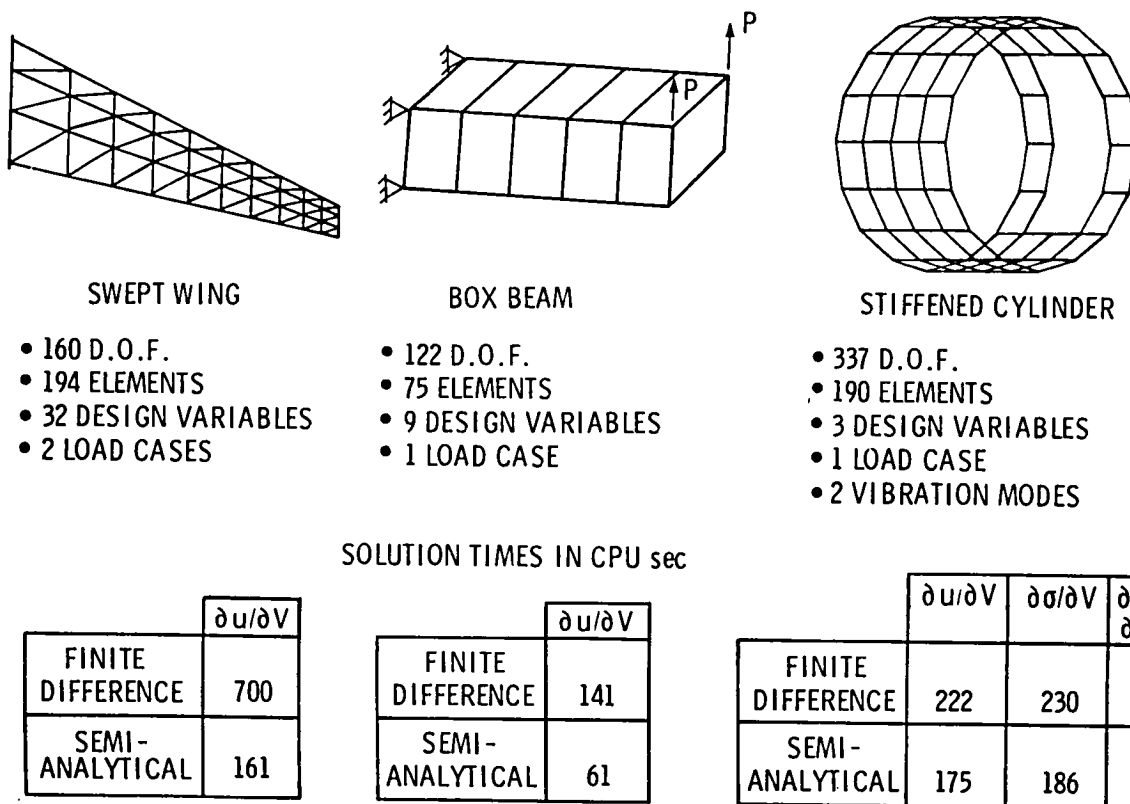


Fig. 7.- Examples of results of a sensitivity analysis.

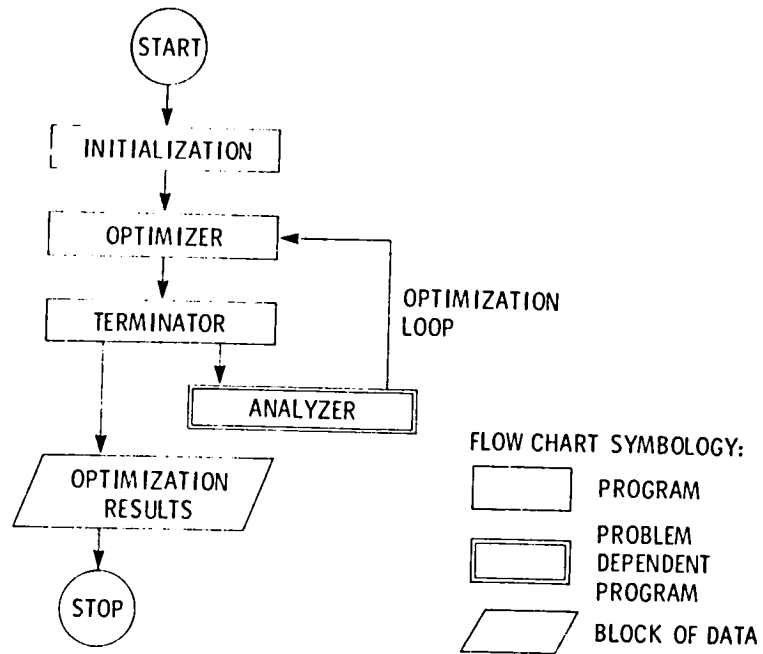


Fig. 4.- Generic components and basic flow organization of an optimization procedure.

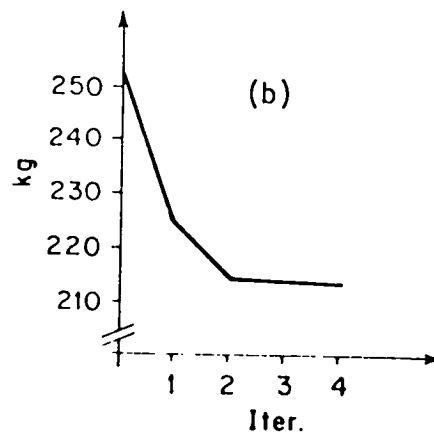
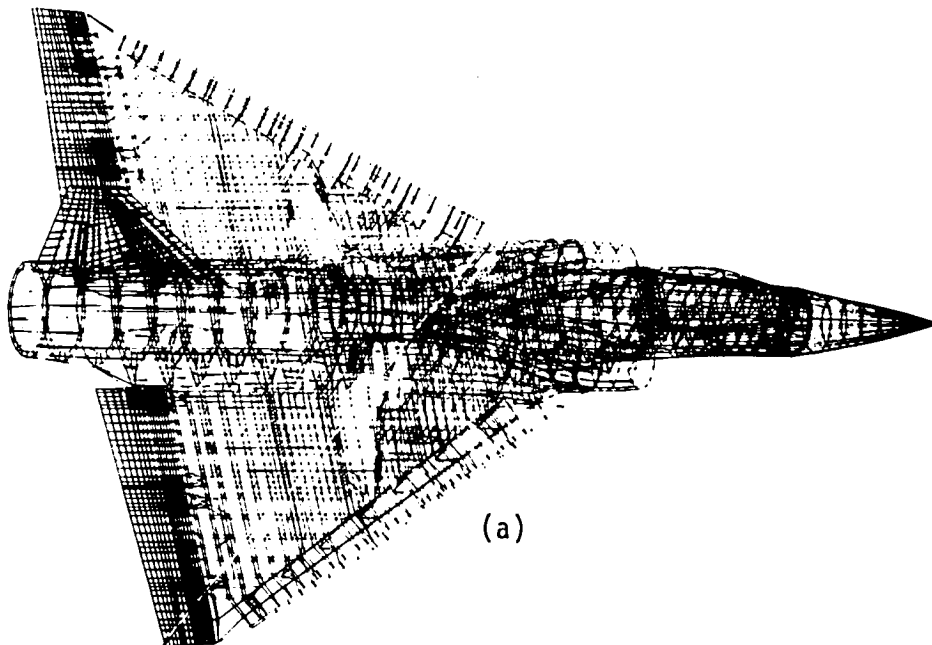


Fig. 5.- Optimization of a complete airframe.

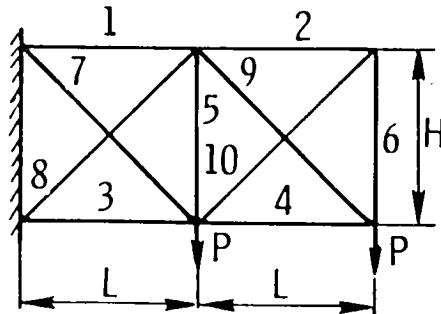


Fig. 8.- A cantilever truss optimized for minimum mass under stress constraints.

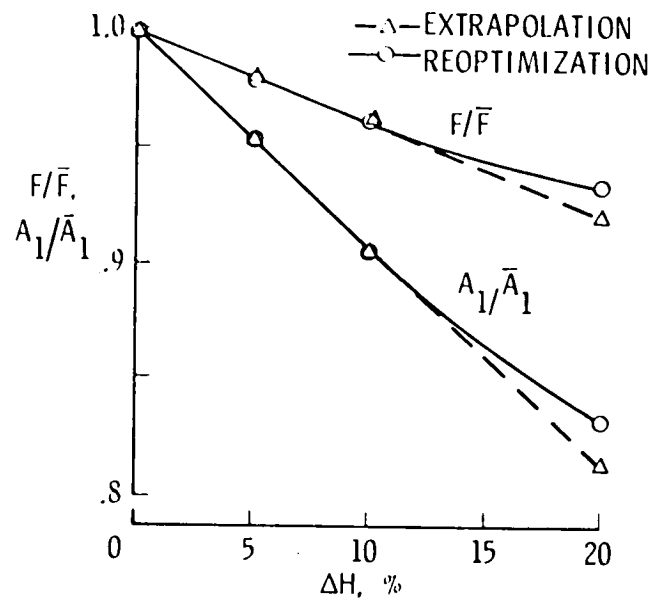


Fig. 9.- Cantilever truss: objective function and one cross section as functions of parameter H.

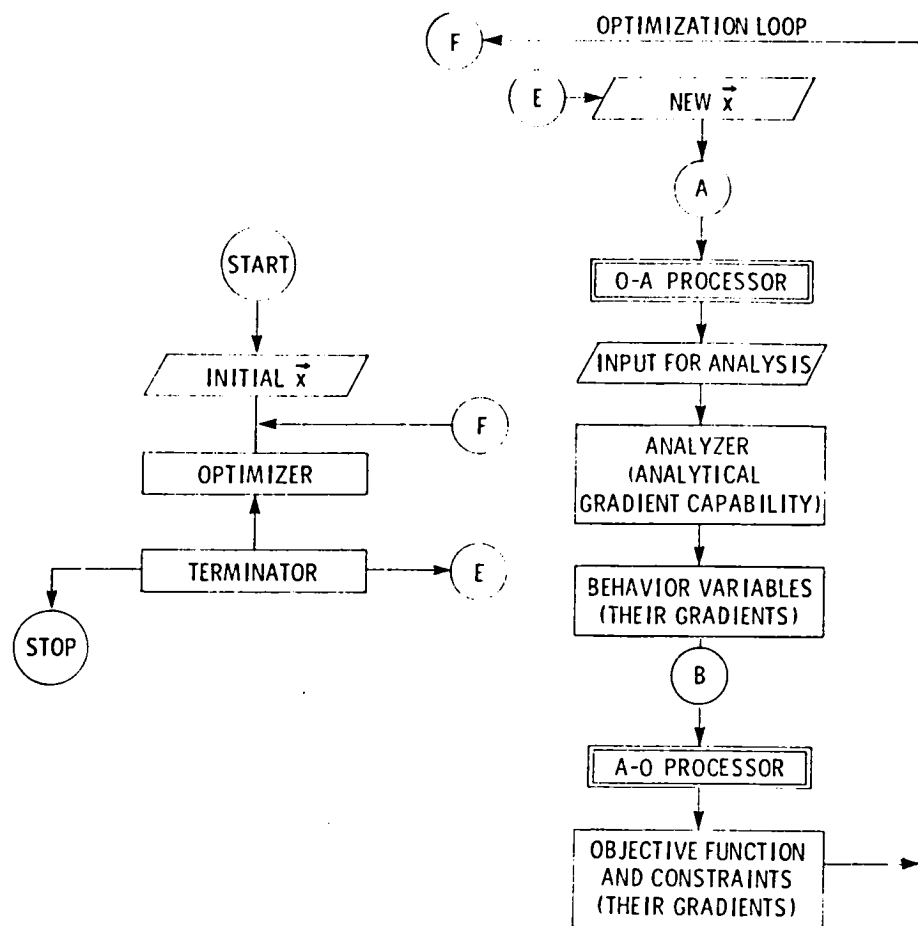


Fig. 10.- Adaptable programing system.

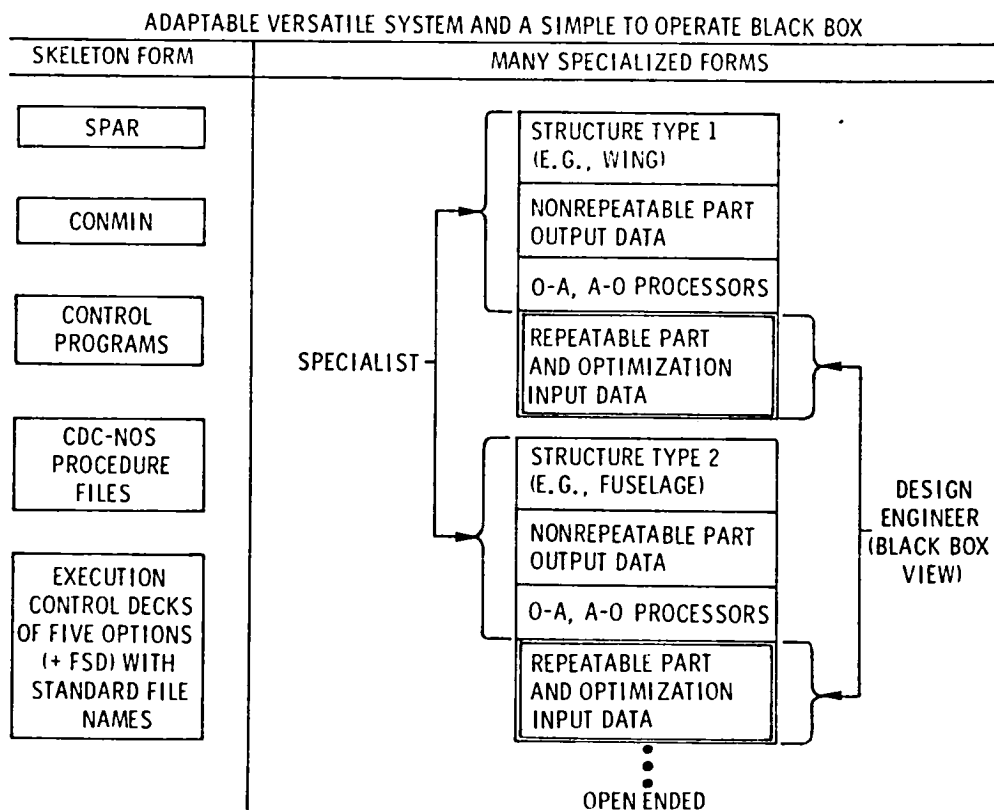


Fig. 11.- Skeleton and specialized forms of a programing system.

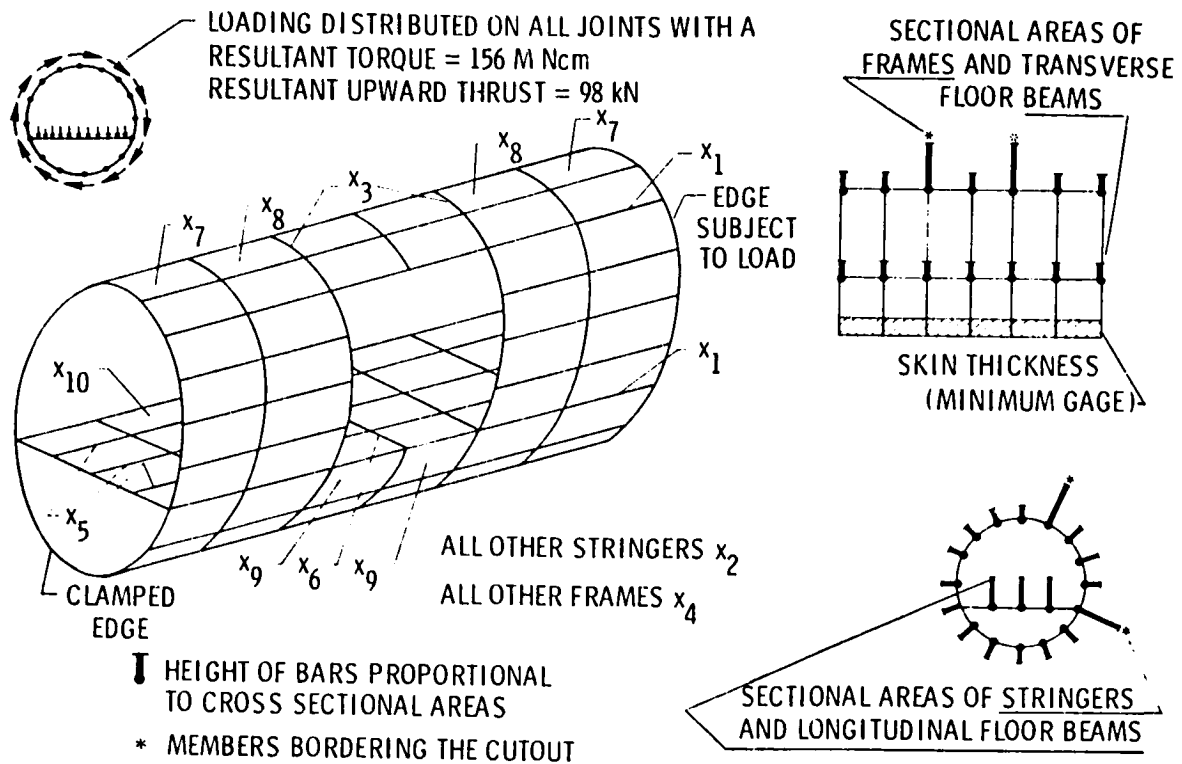


Fig. 12.- Stiffened cylindrical shell optimized for minimum mass.

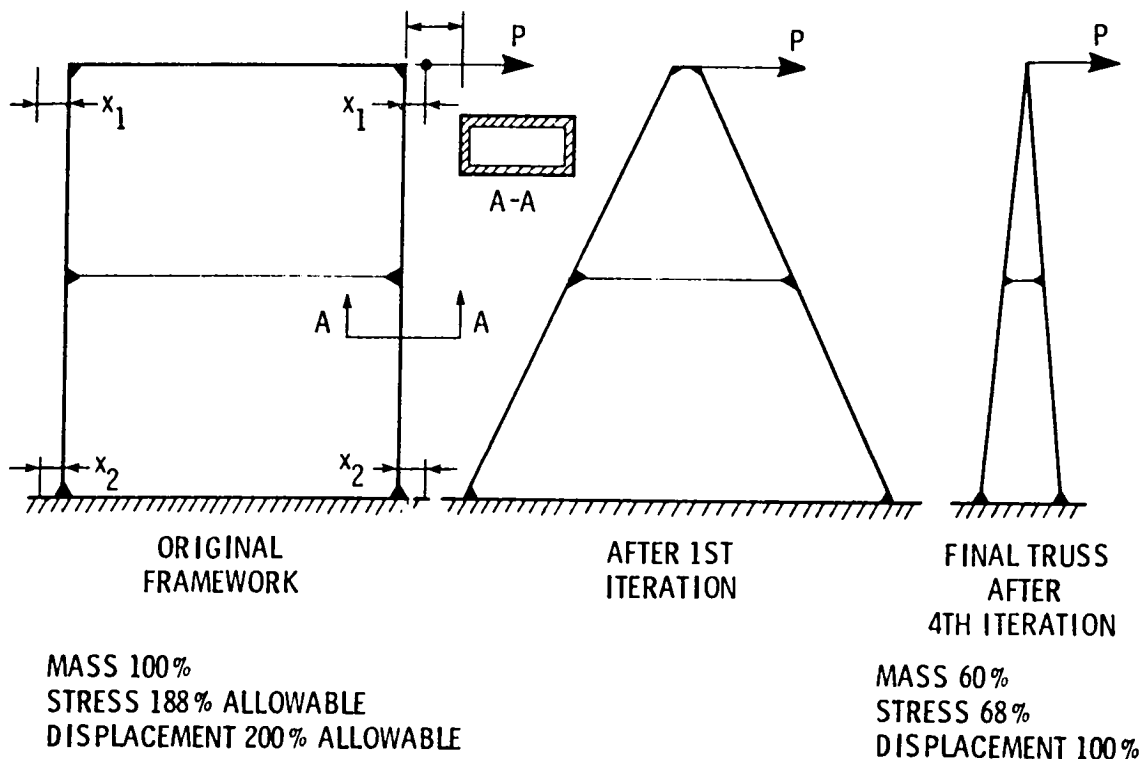


Fig. 13.- Transformation of a framework to a truss by optimization with geometrical variables.

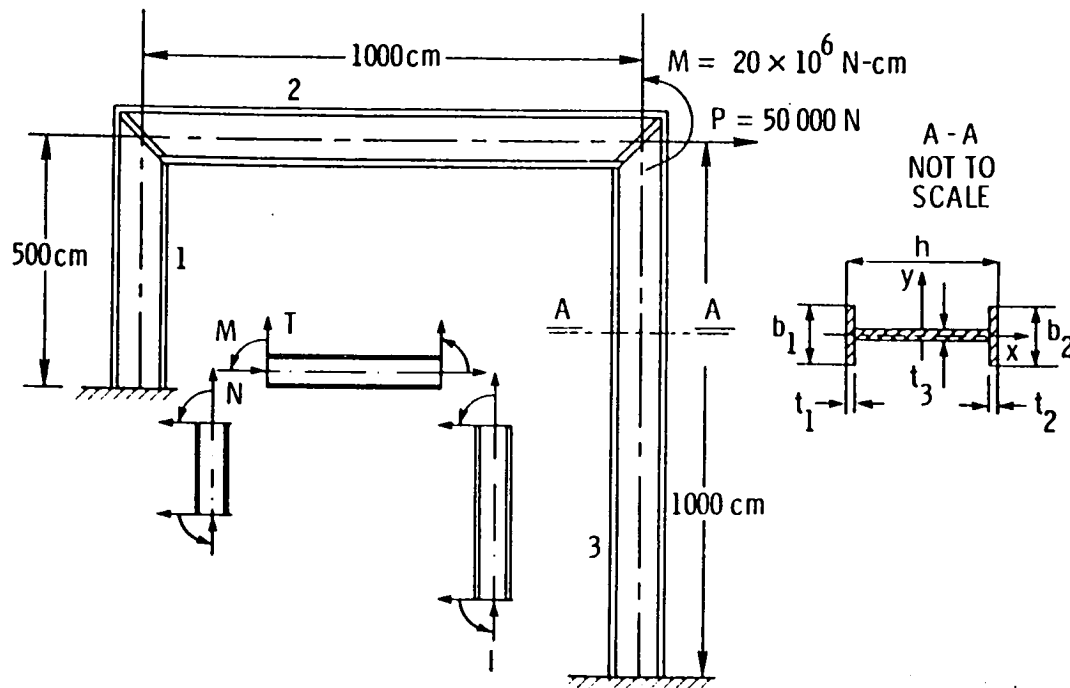


Fig. 14.- Framework as a test case for a two-level optimization procedure.

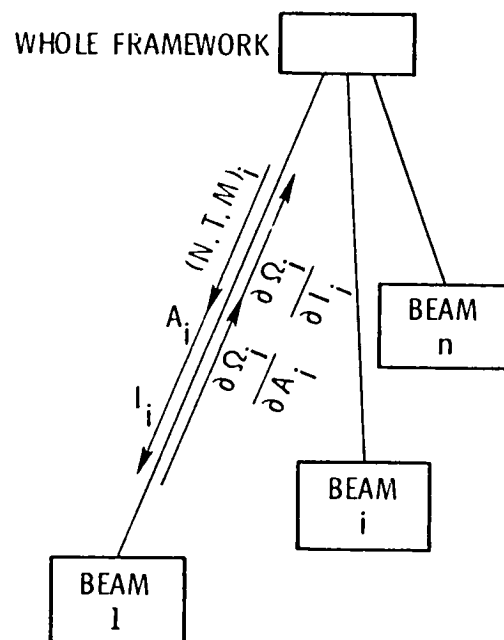


Fig. 15.- Decomposition of a framework into a hierarchy of optimization subproblems.



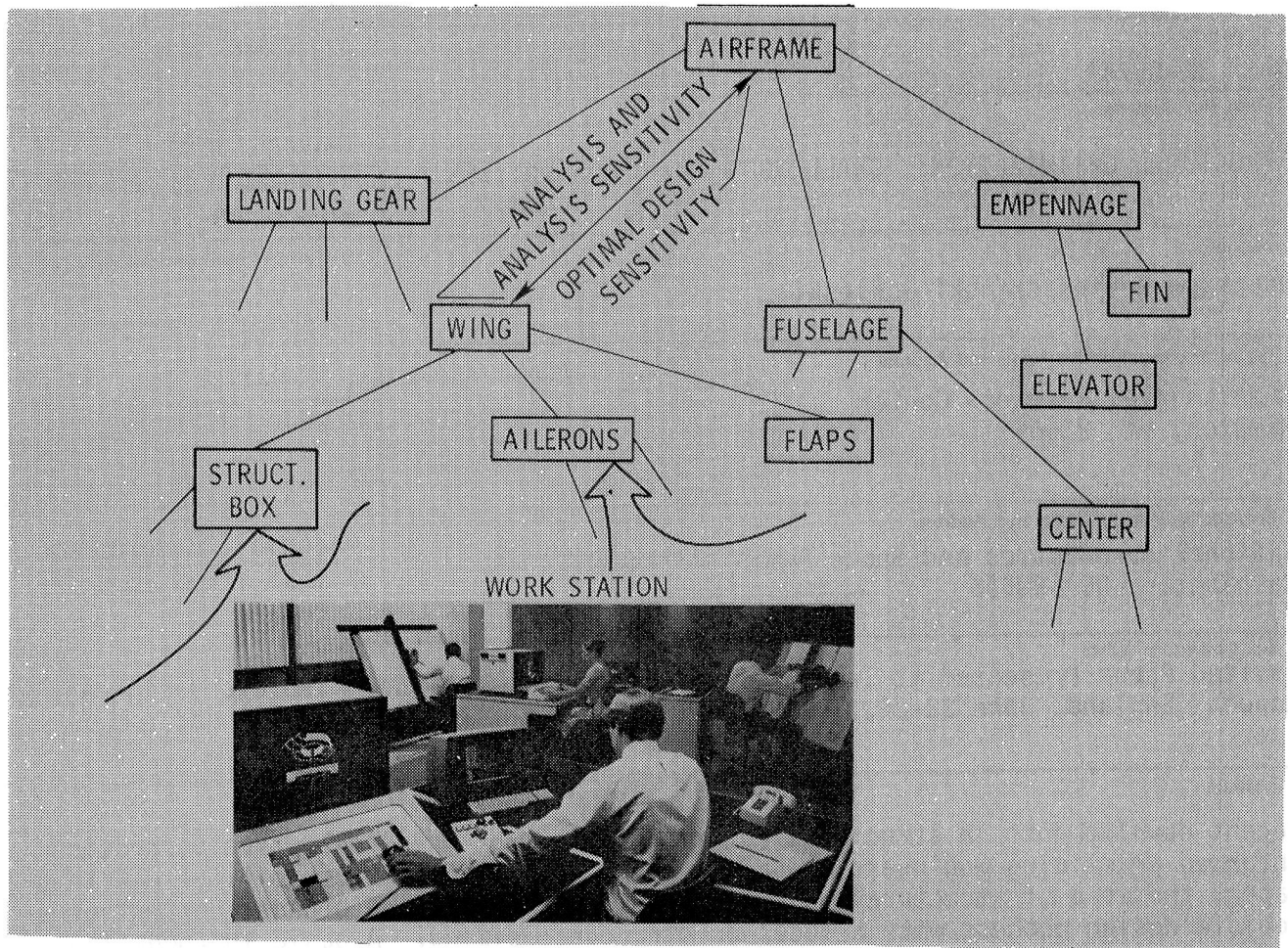


Fig. 16.- Groups of people and computers working concurrently on a large, decomposed design problem.

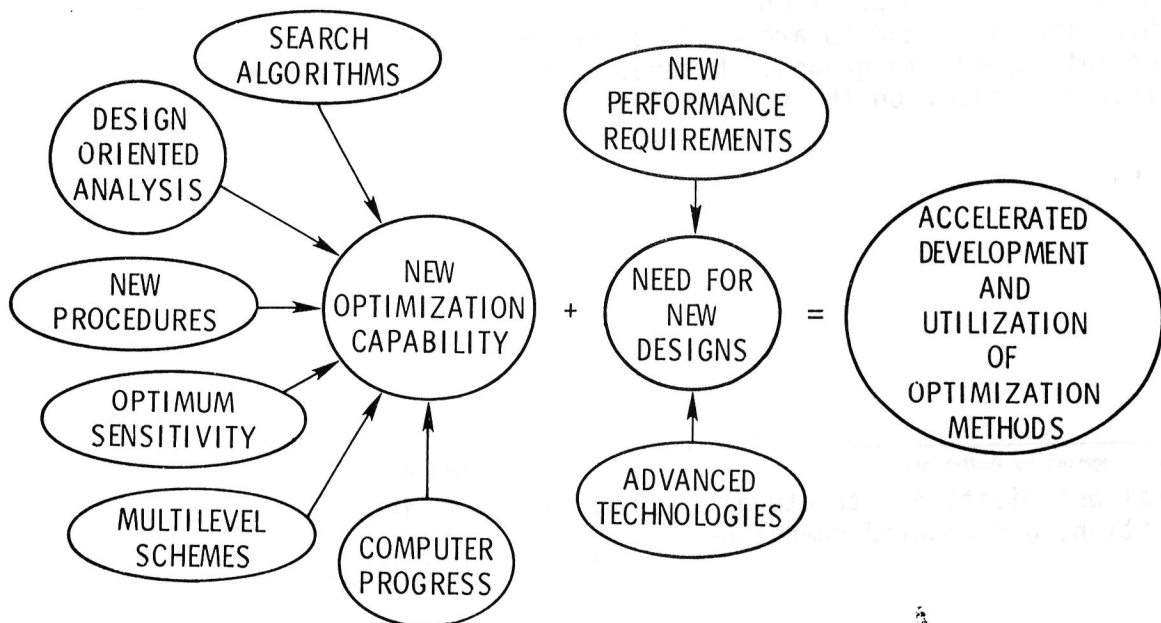


Fig. 17.- Coalescence of the stimuli for further intensified development and use of optimization methods.

1. Report No. NASA TM-85741		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle  STRUCTURAL OPTIMIZATION: CHALLENGES AND OPPORTUNITIES				5. Report Date January 1984	
				6. Performing Organization Code 505-33-53-12	
7. Author(s) Jaroslaw Sobieszczanski-Sobieski				8. Performing Organization Report No.	
				10. Work Unit No.	
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665				11. Contract or Grant No.	
				13. Type of Report and Period Covered Technical Memorandum	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546				14. Sponsoring Agency Code	
15. Supplementary Notes Invited Paper Presented at International Conference on Modern Vehicle Design Analysis, London, England, June 22-24, 1983. Published in Conference Proceedings.					
16. Abstract  Recent developments in structural optimization, when taken collectively, promise informed practicing engineers a quantum jump in their design capability. In this paper, the area of structural optimization is treated in the broader context of a vehicle design process with a focus on structural sizing. A basic introduction to a formal approach is given, and several application examples are illustrated, to lay a background for the review of recent progress. The main developments discussed include techniques for reducing computational cost of optimization, methods for generating sensitivity information, and the ways to make the computer implementations more practical. New prospects are presented for applying optimization to very large problems by formal decomposition into a number of smaller problems in a manner compatible with the trend toward distributed computing for the design process organized into specialty groups. Numerous references are quoted as points of entry to the vast literature on the subject.					
17. Key Words (Suggested by Author(s)) structural optimization, structural sizing, decomposition, distributed computing			18. Distribution Statement Unclassified - Unlimited  Subject Category - <u>05</u>		
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 24	22. Price A02		



